Stephen Elliott August 18, 2006

Details and Methods

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Chapter 2: Plasma Pressure and Thrust 0

Summary: Taking the magnetic field distribution in 2-dimensions and the plasma emission parameters (how many charged particles are emitted, their average charge, and their average mass) find out the distribution of flow at a designated location. Graph the flow distribution parameters. 1

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Chapter 3: Integration of Plasma Throw and Magnetic Field in 3-Dimensions

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Chapter 2 Modelling of a Magnet.

Attempt 1
P. 1

$(\nabla^2 - \mu\epsilon\, \partial^2/\partial t^2)\, \vec{A} = -\mu \vec{J}$

$(\nabla^2 - \mu\epsilon\, \partial^2/\partial t^2)\, V = -\rho/\epsilon$

Units:

$\vec{J}$ amp / meter^2

V volts

Derived units

$\cancel{\vec{A} : (\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2}) V \frac{\epsilon}{\mu c} = -\rho \mu c}$ $\sqrt{\frac{\mu}{\epsilon}} = Z$ ohms

$\cancel{\text{Volt}/(\text{Ohm}\ ^2\ m/s) = \frac{\text{Amp}\cdot s}{\text{Ohm}} = \frac{\text{Coulomb}}{\text{ohm}}}$

$\vec{A} : (\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2})\, V \mu\epsilon c = -\rho c \mu$ $\quad c = \frac{1}{\sqrt{\mu\epsilon}}$

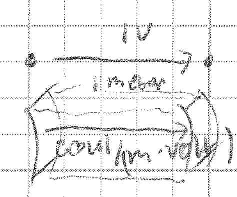
Units $\vec{A} = \frac{\text{Volts}}{m/s}$ Units μ $\frac{\frac{\text{Volts}}{m/s}}{\text{Amp}/m^2} = \text{Ohm}\, \frac{\cancel{m^2}}{\cancel{m^2}}\, \frac{\text{meter}}{1/s}$ meter second

μ Ohm meter second

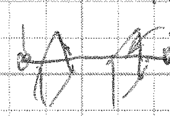
~~Units~~ $\cancel{\epsilon = \frac{\text{Volt}}{\text{coulomb}/m^3}}$

units ϵ $\frac{\cancel{\text{volt}}\ \text{Coulomb}/m^3}{\text{Volt}/m^2}$ volt = Joule/Coulomb

$\cancel{\text{coulomb}/m}$ $\frac{\text{coulomb}}{\text{meter Volt}}$



$D = \epsilon E$ $\quad$ Coulomb/m^2 = ϵ Volt/m



Unit ϵ $\frac{\text{Coulomb}/m^2}{\text{volt}/m}$ $\quad$ unit $\frac{1}{c\epsilon} = \frac{1}{m/s\ \frac{\text{Coulomb}/m^2}{\text{volt}/m}} = \frac{\text{Volt}}{\text{Amp}} = \text{Ohm}$

$\frac{1}{c\epsilon} = \frac{1}{\sqrt{\mu\epsilon}}\,\epsilon^{-1} = \frac{\sqrt{\mu\epsilon}}{\epsilon} = \sqrt{\frac{\mu}{\epsilon}} = Z$

$= 376.7\Omega$ free space

"Charge" perspective

$\cancel{\nabla - \mu\epsilon}$ $(\nabla^2 - \frac{\partial^2}{\partial (ct)^2})\, \vec{V}_4 = -\vec{\rho}_4/\epsilon$

$\vec{V}_4 =\,; -\vec{J}/c\epsilon = (\nabla^2 - \cancel{\mu\epsilon} \frac{\partial^2}{\partial (ct)^2})\, \vec{A}\, \frac{1}{\mu} c\epsilon$ $\quad J$ coul/s·m^2

J/c = coul/m/s s m^2

$\vec{V}_4 =\,; (\nabla^2 - \frac{\partial^2}{\partial (ct)^2})\, \vec{V}_4 = -\vec{\rho}_4/\epsilon$ $\quad$ = coul/m^3

~~where~~ $\cancel{\vec{\rho}_4 = (A_x \frac{\epsilon}{\mu}\epsilon, A_y \frac{\epsilon}{\mu} c, A_z \frac{\epsilon}{\mu} c,}$

where $\vec{\rho}_4 = (J_x/(c\epsilon), \frac{J_y}{c\epsilon}, \frac{J_z}{c\epsilon}, \rho)$

$\cancel{\vec{\rho}_4 = (J_x/Z_0, J_y/Z_0, J_z/Z_0, \rho)}$ $\quad \frac{\cancel{A}\ \text{coul}}{m^2 s}$ $\quad \frac{\text{coul}}{m^3}$

$(\nabla^2 - \mu\epsilon\, \partial^2/\partial t^2)\, \frac{A}{\mu\epsilon} = -J/\epsilon$

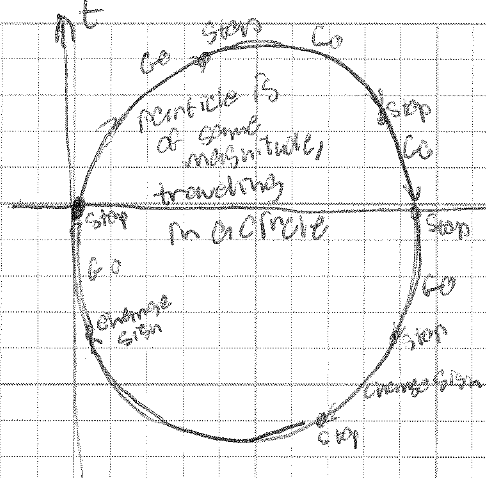
J units $\frac{\text{coul}}{m^2 s} \cdot s/m$ = coul/m^3 $\quad \vec{\square}_4 = (\frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k} + \frac{\partial}{\partial ct}\hat{\tau})$.

$\vec{\rho}_4 = (-J_x/c, -J_y/c, -J_z/c, \rho)$ $\quad$ − sign since $\hat{\tau}\cdot\hat{\tau} = -1$

$\vec{\square}_4 \cdot \vec{\rho}_4 = (\vec{\square}\cdot\vec{J})/c + \frac{\partial}{\partial ct}\rho = 0$ Continuity equation

$\vec{B} = \vec{\nabla}\times\vec{A}$ $\quad \vec{E} = -\vec{\nabla}V - \partial\vec{A}/\partial t$ $\quad \vec{\square}_4 \cdot \vec{\square}_4 = (\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2})$

$\hat{\tau}\cdot\hat{\tau} = -1$



$$\vec{\Box}_4 \circ \vec{\Box}_4 \vec{V}_4 = -\frac{1}{\epsilon}\vec{\rho}_4$$

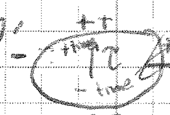
$$\left(\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2}\right)\vec{V}_4 = -\frac{1}{\epsilon}\left(\frac{J_x}{c}, \frac{J_y}{c}, \frac{J_z}{c}, \rho\right)$$

$$\vec{\Box}_4 \bullet \vec{\rho}_4 = 0 = \nabla \circ \frac{\vec{J}}{c} + \frac{\partial \rho}{\partial ct} \quad \text{continuity}$$

$$\Rightarrow \vec{\rho}_4 = \vec{\Box}_4 \times \vec{\rho}_{4A}$$

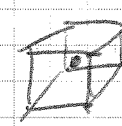
$\longrightarrow J_x$

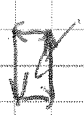
charge is a directed quantity.



No net charge, just a distribution

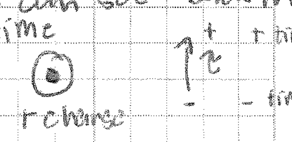
Application to a magnet in 2-D:



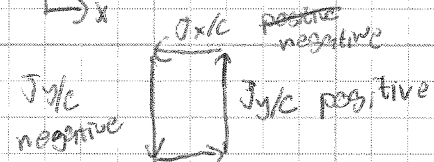


directed charge moving in a direction we can model it as having a constant conventional velocity that changes its orientation on its path.

We can see back in time

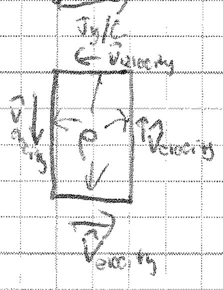


$\rho V/c$



Since $|\vec{J}/c|$ is constant and the orientation simply changes along its path, the charge along the path can be represented as

$$\rho V/c \, \hat{v} = \rho \hat{v}$$



The concept can be extended to $\hat{v}_4$ where most of the modeled "moving" charge is oriented in (x, y, z) and 0 in $\vec{\tau}$ direction.

I do not need to concern myself with charge dynamics in terms of mass dynamics internally for the simple model.

Since ρ is assumed to have equal ~~area~~ extent here in all 4 directions the variation in velocity ~~accounts for~~ orientation accounts for the different flow. ??

$\cancel{\vec{V}_4 = c\,\hat{v}}$ $\quad \vec{S}_4 = c\,\hat{S}_4$

$\hat{v}$ is confusing for velocity since it is used for potential already use $\vec{S}_4$

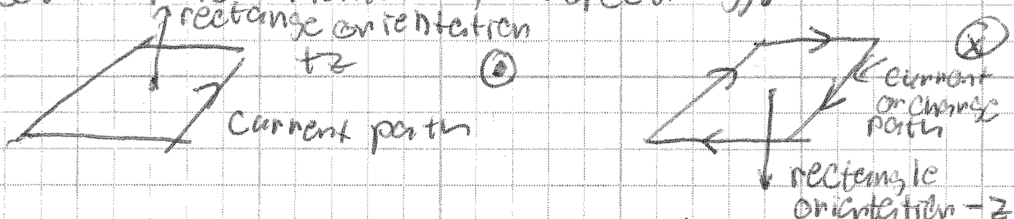
That means that the electricity in this model travels with the speed of light, and the magnitude of ρ accounts for variations in amount of charge flowing

When accounting for the motion of each electron, it may be that an electron is at rest, moving at a speed less than light, or there may be charge positively oriented. However, this setting of $v_{elect} = c$ speed of light is a field perspective. The electrodynamics are in going between equilibrium between the field and charge perspectives. In the field perspective, light travels at the speed of light in free space. The charge being modeled here is the radiated charge.

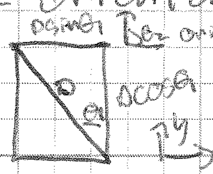
Magnet Model:

A magnet is a series of points that make up the current path. ~~Ato~~ The moving charge that contributes to radiation is ρ and that charge moves at the speed of light. The medium is taken as free space.

A bar magnet has a ~~path~~ current path in a rectangle oriented in a direction. For the 2-d model, that orientation is in the posititive or negative z direction (north or south pole orientation, respectively).

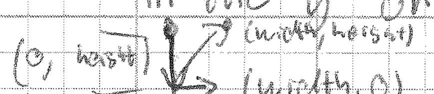


The rectangle has width, ~~amt~~ height, and ~~ple~~ clock orientation



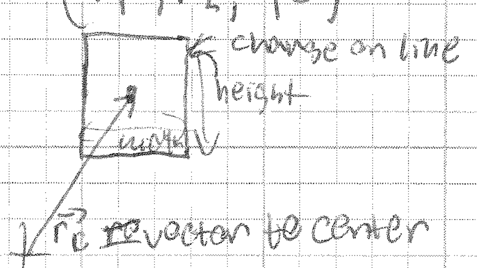
The width/height is the aspect ratio

I am orienting all the modelled rectangle in the y orientation



So each magnet is measured by three parameters

~~($\vec{r_1}$, $\vec{r_2}$, ρ)~~ ($\vec{r_1}$, width, height, charge)



An arc magnet also has a width, length, and position to it. However, the length is best characterized by a radius vector, a start angle, and a finish angle. For a quarter circle arc magnet, the finish angle is 45° greater than the start angle. In the simple model, the start angle can be 0°, 90°, 180°, or 270°.

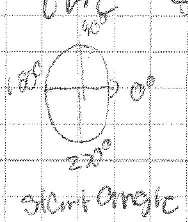
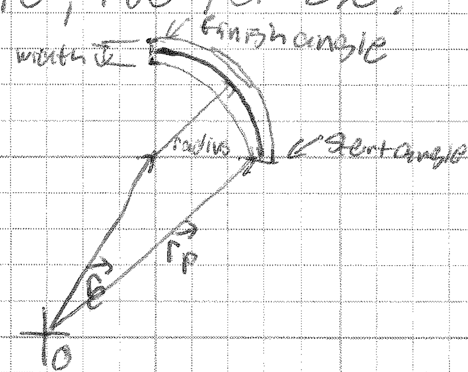


Fig 2-4.1 Start Angle



The ~~quarter circle arch~~ arc magnet can have a defined standard radius, and width, as in the case of a bar ~~mag~~ magnet. The bar magnet can be of two varietes, ~~vertical bar magnet~~ and horizontal bar magnet where the widths and heights are defined for the two types. Similarly, the quarter circle arc magnet can be of the 0°, 90°, 180°, or 270° variety according to its start angle.

On a closed path with rounds, each step ~~of~~ of position advances according to the path length addition corresponding to the path vector $\vec{r}_p$.

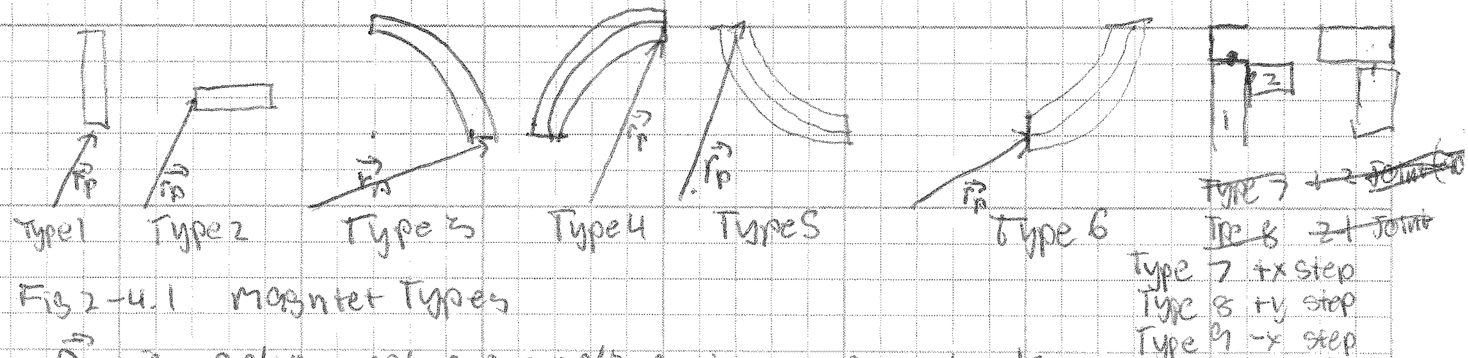


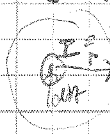
Fig 2-4.1 Magnet Types

$\vec{r}_p$ is advanced according to magnet type

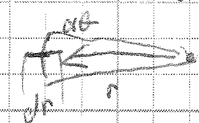
The steps do not have any magnet strength. Rather the

In statics

$\nabla^2 \vec{A} = -\mu \vec{J}$, $\vec{B} = \vec{\nabla} \times \vec{A}$ $= \int \vec{B}\, dl = \int \vec{\nabla} \times \vec{A}\, dl = \oint \hat{n} \times \vec{A} \Big|_a^b$



$B = \dfrac{\mu I}{2\pi r}$

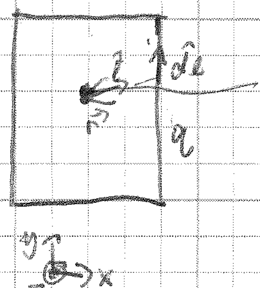


$J\,dA = I$
$= J\,dr\,d\theta$

$d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{I\, d\vec{l} \times \hat{r}}{r^2}$ Biot Savart

$= \dfrac{\mu_0 I}{2r} \left(\dfrac{d\vec{l}}{2\pi r} \right) \times \hat{r} = \dfrac{\mu_0 I}{2r} d\vec{l} \times \hat{r}\, d\theta$ Let $\vec{\theta} = \dfrac{\vec{l}}{\cancel{d}2\pi r} \Rightarrow d\vec{\theta} = \dfrac{d\vec{l}}{2\pi r}$

The strength of the magnet can be determined by its field strength ~~a~~ measured at a point



test point field strength for a bar magnet

$d\hat{l} \times \hat{r}$ is ~~$\hat{z}$~~ in $\hat{z}$ direction, so can come back to scalar orientations

B_z is the scalar

A general parameter can be line divisions how many segments each line or curve is devided into in order to make the necessary calculation.

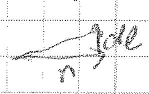
The magnet thus can be characterized in how many Gauss strong it is at a point so that all parameters are in Gauss

Better yet, set all parameters in Tesla which is the SI standard unit for B-field.

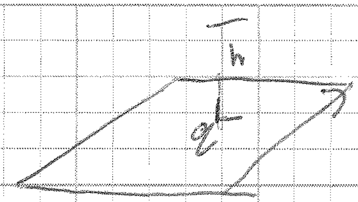
1 Tesla is 10^4 Gauss
1 Tesla $= 1\ V \cdot s/m^2 = \dfrac{1\ V/m}{m/s}$
$B = \mu H$
$\mu = 4\pi \times 10^{-7}\ \cancel{A/m}\ \dfrac{H}{m}$ (~~inductance~~ magnetic permeability)
$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial}{\partial t} \vec{D}$

First programming objective is to determine q ~~for a~~ for a bar magnet based on the measurements at three points?

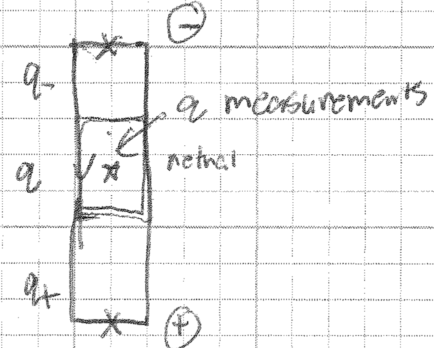
For simplicity, start with center point calculation
~~It~~ Should scale linearly.



Reference :
1) wikipedia "Biot Savart law", "Earth's Magnetic Field" "Orders of Magnitude (magnetic field)",



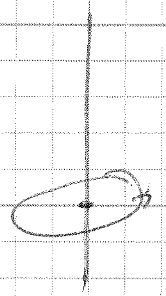
Standard height.
Each measurement is expected to give a different value for q. The question is how to quantify it.
To do so simply ther are different q values dependend on where measured
More complex, ~~the~~ introduce "h" the distance of measurement



Better yet, have 3 "virtual" magnets per physical bar magnet to account for domain variation within the magnet.

There will be an average q for the whole magnet and variations + and −.

Simplify the magnet model even further. Since most of the perimeter is made of the edges, take the field at the edge as contributions



$B = \frac{\mu I}{2\pi r}$ for top and bottom

If just top

$B = \frac{\mu I}{4\pi r}$

middle w $\longrightarrow$ $B = \frac{\mu I}{2\pi(\frac{w}{2})} + \frac{\mu I}{2\pi(\frac{w}{2})} = \frac{4 \mu c q}{2\pi w}$

end $B = \frac{2 \mu c}{2\pi}\left(\frac{q}{w}\right) \rightarrow \Downarrow \Uparrow$

So in the middle, the q contribution is half what it is at the ends for a given B field

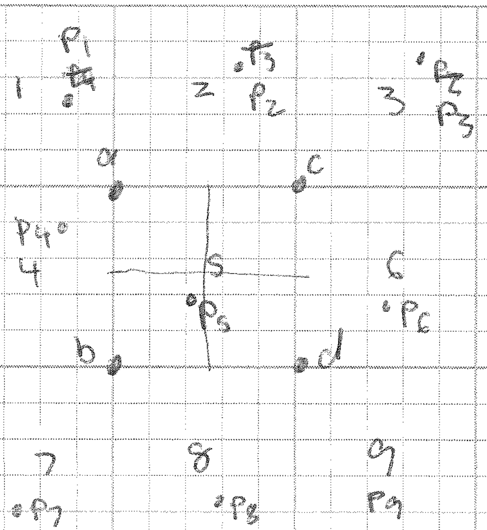
Since B is measured in Tesla, alternatively $q = Bw\left(\frac{\pi}{\mu c}\right)$ end

$\left(\frac{\pi}{\mu c}\right)$ is a conversion factor, $Bw \, 2\left(\frac{\pi}{\mu c}\right)$ center

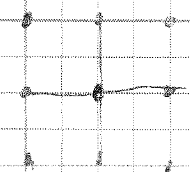
If w is uniform, I can then look at the B contribution as a point source. Or if I want a line source model, I can convert to the "q" model on the edges.

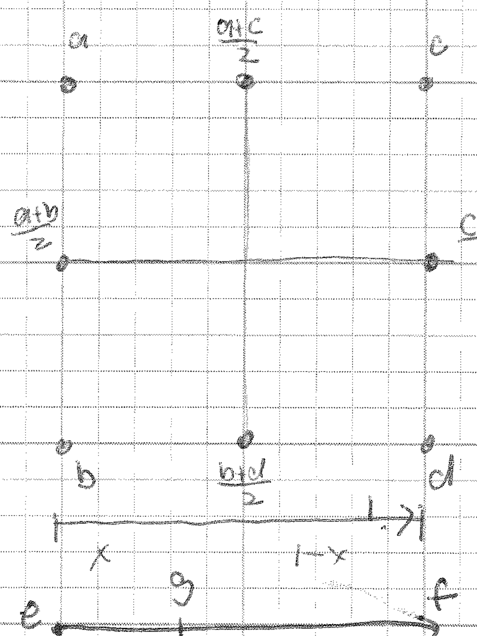
I like the edge line-segment model the best.

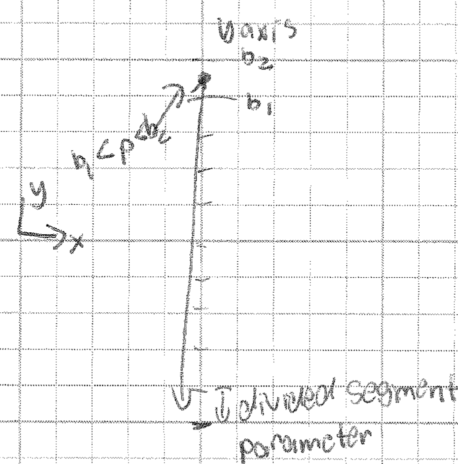
Contour line drawing

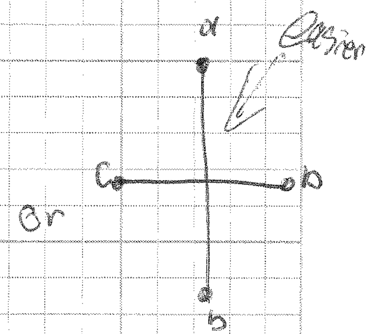


The contour point may or may not exist within the boundaries shown as (a, b, c, d) which is at left. But if so, its location can be determined









$g = \cancel{xe + (1-x)f} = xf + (1-x)e = x(f-e) + e$

$x = \dfrac{g-e}{f-e} = \cancel{g+f}\ \dfrac{g-f+f-e}{f-e}$

~~step~~

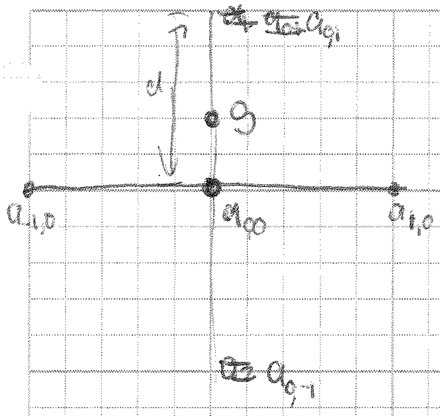
step 1: Is $e<g<f$? Set point at origin

step 2: Is $\frac{1}{f-e}$ a number (Is it too big?)

yes: stop calculation

no: proceed to calculate x for each line segment

Contour line drawing



$$\frac{\Delta a}{\Delta y} = \frac{a_{0,1} - a_{0,-1}}{2d}$$

bounds $a_{0,0} \pm d \times \frac{\Delta a}{\Delta y}$

$g - a_{00}$

~~One very simple~~

Δa has to exceed a minimum value or else the contour point is set at the origin

One very simple way of doing the contour is if the x and y values are in range, then the square is dark. I like this best. That means rectangles are drawn. The plot area is a grid point set.

Another approach to contour plot.
Assign a grid with each square having a value. Then set a contour band. If the value of the square is within the band, it darkens.

Chapter 1 Math for Magnet Model

$$\nabla\cdot D = \rho \qquad \nabla\cdot B = 0 \qquad \vec\nabla\times\vec E = -\frac{\partial \vec B}{\partial t} \qquad \vec\nabla\times\vec H = \vec J + \frac{\partial}{\partial t}\vec D$$

$$\vec D = \epsilon\vec E \qquad \vec B = \mu\vec H$$

$$\vec\nabla\times\vec\nabla\times\frac{\vec B}{\mu} = \vec\nabla\times\vec J + \frac{\partial}{\partial t}\vec\nabla\times\epsilon\vec E = \vec\nabla\times\vec J - \epsilon\frac{\partial^2}{\partial t^2}\vec B$$

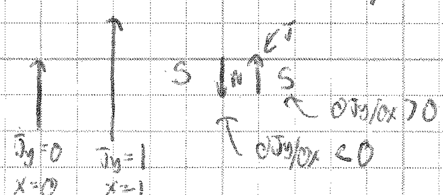
$$-\nabla^2\vec B = \mu\vec\nabla\times\vec J - \mu\epsilon\frac{\partial^2}{\partial t^2}\vec B$$

$$\left(\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2}\right)\vec B = -\mu\vec\nabla\times\vec J$$

$$\vec J = J_x\hat i + J_y\hat j$$

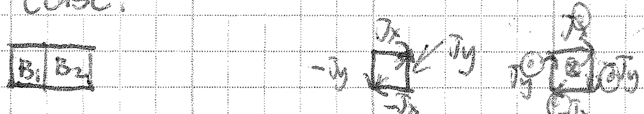
$$\vec\nabla\times\vec J = \begin{vmatrix} \hat i & \hat j & \hat k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ J_x & J_y & \cancel{J_z}^{\,0} \end{vmatrix} = \hat k\left(\frac{\partial J_y}{\partial x} - \frac{\partial J_x}{\partial y}\right) - \hat i\frac{\partial J_y}{\partial z} + \hat j\frac{\partial J_x}{\partial y}$$

$$\left(\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2}\right)B_z = -\mu\hat k\left(\frac{\partial J_y}{\partial x} - \frac{\partial J_x}{\partial y}\right) = \mu\hat k\left(\frac{\partial J_x}{\partial y} - \frac{\partial J_y}{\partial x}\right)$$

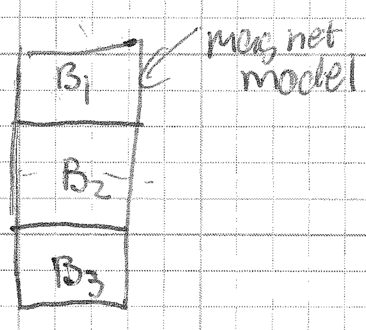


If the model for B_z is ~~tE~~ linear precewise on the grid then $\partial J_x/\partial y = \nabla J_x/\nabla y$. Let us approach the statics case.

$\vec B = B\hat z$

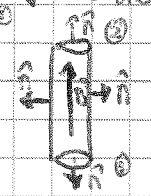


So it is equivalent then to have a surface with $\vec B$ field on a grid and then calculate $\vec J$ after (if $\vec J$ is even needed).



$\vec{\nabla}\times\vec{E} + \partial\vec{B}/\partial t = 0$ $\quad \vec{\nabla}\times\vec{H} - \frac{\partial}{\partial t}\vec{D} = \vec{J}$ $\quad \vec{D} = \epsilon\vec{E}$ $\quad \vec{B} = \mu\vec{H}$ $\quad \vec{\nabla}\cdot\vec{B} = 0$

$\oint_{\odot}^{\odot}\vec{\nabla}\times\vec{B}\,dl = \hat{n}\times\vec{B}\Big|_{\odot}^{\odot}$, Therefore $\odot\vec{B} = \vec{B}_{\odot} = 0 = \hat{n}\cdot\vec{B}\Big|_{\odot}^{\odot}\hat{b}$



$\otimes\vec{B} = \vec{B} - \odot\vec{B} = \vec{B}_{\otimes}$

$\vec{B} = \vec{\nabla}\times\vec{A}$ $\quad \vec{\nabla}\times\vec{\nabla}\times\vec{A} = -\nabla^2\vec{A} + \vec{\nabla}\vec{\nabla}\cdot\vec{A}$

$\vec{A} = \vec{A}_{\odot} + \vec{A}_{\otimes}$ where $\otimes\vec{A} = \vec{A}_{\otimes}$ and $\odot\vec{A} = \vec{A}_{\odot}$

$\vec{\nabla}\times\mu\vec{H} = \mu(\vec{J} + \frac{\partial}{\partial t}\vec{D})$ $\quad \mu$ a const

$= \mu(\vec{J} + \frac{\partial}{\partial t}\epsilon\vec{E})$ $\quad \epsilon$ const

$\vec{\nabla}\times\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = -\nabla^2\vec{A} + \vec{\nabla}\vec{\nabla}\cdot\vec{A} = \vec{\nabla}\times\mu(\vec{J} + \frac{\partial}{\partial t}\epsilon\vec{E})$

$\vec{\nabla}\times(-\nabla^2\vec{A} + \vec{\nabla}\vec{\nabla}\cdot\vec{A}) = \vec{\nabla}\times\mu\vec{J} - \mu\epsilon\frac{\partial^2}{\partial t^2}\vec{\nabla}\times\vec{A}$

$(\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2})\vec{\nabla}\times\vec{A} = -\mu\vec{\nabla}\times\vec{J} \Rightarrow (\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2})\vec{A}_{\otimes} = -\mu\vec{J}_{\otimes}$

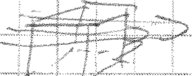
what about $\vec{A}_{\odot}$? to make it convenient to calculate in other coordinate systems:

$(\nabla^2 - \mu\epsilon\frac{\partial^2}{\partial t^2})\vec{A}_{\odot} = -\mu\vec{J}_{\odot}$

$\vec{\nabla}\cdot(\vec{J} + \frac{\partial}{\partial t}\vec{E}) = \vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$

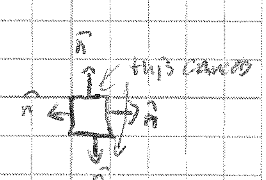
$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$ $\quad (\vec{\nabla}\times, \frac{\partial}{\partial t})\cdot(-\frac{\partial}{\partial t}, \vec{\nabla}\times)\vec{A}_{\otimes} = 0$ where $\begin{bmatrix}\vec{E}_{\otimes}\\ \vec{B}_{\otimes}\end{bmatrix} = \begin{bmatrix}-\frac{\partial}{\partial t}\\ \vec{\nabla}\times\end{bmatrix}\vec{A}_{\otimes}$

For H, $E_{\odot}$ is due to the central proton for any volume element that includes the proton. It is important to define the volume element. Outside, that field shows as $\vec{E}_{\otimes}$.



Example: Cylindrical coordinates, central proton

$\vec{E}_{\otimes} = \frac{1}{4\pi\epsilon}\frac{1}{r^2}\hat{r}$?, volume in $r>0$



this causes

$\int\hat{n}\cdot\vec{E}_{\otimes}\,dl = 0!$, $\oint\hat{n}\times\vec{E}_{\otimes}\,dl = 0!$

$\vec{E}_{\odot}$ only if charge inside!

If $\vec{A}_{\otimes}$ is only measuring radiation, then what happens in a rotating frame of reference? If $\vec{A}_{\otimes}$ is locally a plane wave, $\vec{A}_{\otimes} = \vec{A}_0 e^{j(\omega t + \vec{k}\cdot\vec{r})}$

$-k^2 + (\frac{\omega}{c})^2 = 0 \Rightarrow |k| = \frac{\omega}{c}$, $\vec{A}_{\otimes} = \hat{A}_{\otimes}e^{j(|k|(ct + \hat{n}\cdot\vec{r}))}$

$\frac{d\vec{A}_{\otimes}}{d(ct)} = \cancel{A_{\otimes}}(\frac{d}{d(ct)}\hat{A}_{\otimes})e^{j(|k|(ct + \hat{n}\cdot\vec{r}))} + \hat{A}_{\otimes}j|k|(1 + \vec{v}/c)e^{j(|k|(ct + \hat{n}\cdot\vec{r}))}$

$\frac{\partial}{\partial t}(\omega t + \vec{k}\cdot\vec{r}) = |k|(1 + \hat{n}\cdot\vec{v}/c)$

$\vec{\nabla}\times A_{\otimes}\hat{A}_{\otimes} = (\vec{\nabla}A_{\otimes})\times\hat{A}_{\otimes} + A_{\otimes}\vec{\nabla}\times\hat{A}_{\otimes}$

So what about $\vec{\nabla}\times\hat{A}_{\otimes}$?

$(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z})\times A_{\otimes}$

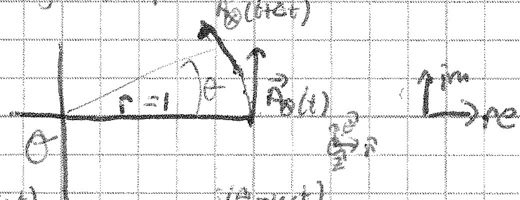
$= (\vec{\nabla}A_{\otimes})\times$

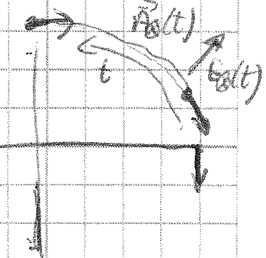
$\vec{\nabla}\times(\vec{A}_{\otimes}\vec{B}) = \vec{B}\,\vec{\nabla}\times\vec{A}_{\otimes} + \vec{A}_{\otimes}\vec{\nabla}\times\vec{B}$

$\vec{\nabla}\times(\hat{A}\hat{B}) = \hat{B}\,\vec{\nabla}\times\hat{A} + A\,\vec{\nabla}\times\hat{B}$ $\quad$ If $\hat{B} = \nabla\varphi$, $\vec{\nabla}\times\hat{B} = 0$

If $\hat{B}\perp\vec{\nabla}A$, $|(\vec{\nabla}A)\times\hat{B}| = |\vec{\nabla}A| \Rightarrow |\vec{\nabla}\times(A\hat{B})| = |\vec{\nabla}A|$

Example, second attempt. What $\vec{A}$ field would be needed to create an E-field that is directed radially out?





$$\begin{bmatrix}\vec{E}_\otimes\\ \vec{B}_\otimes\end{bmatrix} = \begin{bmatrix}-\frac{\partial}{\partial t}\\ \vec{\nabla}\times\end{bmatrix}\vec{A}_\otimes$$

$$-\frac{\partial}{\partial t}\left(j e^{j(\theta-\omega t)}\right) = j j\omega e^{j(\theta-\omega t)} = +\omega e^{j(\theta-\omega t)}$$

So if $\vec{A}_\otimes(x,y) = +\sin(\theta-\omega t)\hat{x} - \cos(\theta-\omega t)\hat{y} = -\hat{\theta}(\theta-\omega t)$

$$\vec{E}_\otimes = -\frac{\partial}{\partial t}\vec{A}_\otimes(x,y) = \left(+\cos(\theta-\omega t)\hat{x} + \sin(\theta-\omega t)\hat{y}\right)(+\omega) = \omega\hat{r}(\theta-\omega t)$$

For $\vec{E}_\otimes = \frac{1}{4\pi\epsilon_0}\frac{e_c}{r^2}\hat{r}$, ~~$\vec{A}_\otimes(x,y)$~~ $\vec{A}_\otimes(\theta,t) = \frac{1}{4\pi\epsilon_0}\frac{e_c}{r^2\omega}\hat{\theta}(\theta-\omega t)$

Must have the point source rotate in order for $\vec{E}_\otimes$ to always point ~~in~~ out.

$$\vec{\nabla}\times\vec{A}_\otimes = \left(\hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta}\right)\times\frac{1}{4\pi\epsilon_0}\frac{e_c}{r^2\omega}\hat{\theta}(\theta-\omega t) = \frac{1}{4\pi\epsilon_0}\frac{e_c}{r^2\omega}\frac{1}{r}\hat{z} = \vec{B}_\otimes$$

$\vec{F}_\perp = \gamma q(\vec{E}_\otimes + \vec{v}\times\vec{B}_\otimes)$ For electron $\vec{F}_\perp = \gamma m_0\frac{\partial\vec{v}}{\partial\theta}\frac{\partial\theta}{\partial t} = \gamma m_0\omega r\frac{\partial\hat{\theta}}{\partial\theta} = -\gamma m_0\omega^2 r\hat{r}$

$$\vec{F}_\perp = \gamma(-e_c)\left(\frac{1}{4\pi\epsilon_0}\frac{e_c}{r^2}\hat{r} + \omega r\hat{\theta}\times\frac{1}{4\pi\epsilon_0}\frac{e_c}{r^2\omega}\frac{1}{r}\hat{z}\right) = -\gamma 2\frac{e_c^2}{4\pi\epsilon_0}\frac{1}{r^2}\hat{r}$$

$$-\gamma m_0\omega^2 r = -\gamma\frac{e_c^2}{4\pi\epsilon_0 r^2}2 \Rightarrow \frac{1}{2}\omega^2 r^3 = \frac{e_c^2}{2\pi m_0\epsilon_0} = \frac{1}{2}\frac{e_c^2}{4\pi m_0\epsilon_0}$$

Because $\vec{A}_\otimes$ needs to vary in time so that the E-field points inward, being parallel to the particle path, that means that means that in the rest frame θ^* there is a magnetic field induced (since both $E_\otimes$ and $\vec{B}_\otimes$ are related to $A_\otimes$). It is not possible to apply an electric field radially without a corresponding magnetic field.

$$(\pi r^2)(\omega^2 r) = \frac{1}{2}\frac{e_c^2}{m_0\epsilon_0} = (\text{Area})\times(\text{Electron Acceleration})$$

$$\frac{1}{2}m_0 v^2 = \frac{e_c^2}{4\pi\epsilon_0}\frac{1}{r}$$ classical Kinetic Energy + Potential Energy = 0

$\frac{\omega}{B} = c$

~~$\vec{p} = \hbar\vec{k} = \hbar\frac{\vec{\omega}}{c} = mv$~~ $p = \hbar k = \hbar n\frac{2\pi}{2\pi r} = mv \Rightarrow v = \frac{\hbar}{m}\frac{1}{r}$

$\Rightarrow$ This gets the condition to solve for the travel path in the Bohr model.

Thomas Precession (summary of published paper)

Magnetic moment of an electron:

$\vec{\tau} = \nabla \cdot \vec{v} \times \vec{B}$

$$\vec{\mu} = \frac{ge}{2mc}\vec{S} \quad , \quad \vec{S} = \pm\frac{\hbar}{2}\hat{s}$$

$$U_{AZ} = \frac{-ge}{2mc}\vec{S}\cdot\vec{B} \quad \text{with } g=2$$

Splitting with applied magnetic field
Anomalous Zeeman effect

$$U_{SO} = \frac{g}{2m^2c^2}(\vec{S}\cdot\vec{L})\frac{1}{r}\frac{dV}{dr} \quad \text{with } g=1$$

Normal Zeeman effect

$$\left(\frac{d\vec{S}}{dt}\right)_{\text{rest frame}} = \vec{\mu}\times\vec{B}'$$

$\vec{B}'$ magnetic field

$$\vec{B}' = \gamma\left(\vec{B} - \frac{\vec{v}}{c}\times\vec{E}\right)$$

$$\left(\frac{d\vec{S}}{dt}\right)_{\text{rest frame}} = \vec{\mu}\times\vec{B}' = \vec{\mu}\times\gamma\left(\vec{B} - \frac{\vec{v}}{c}\times\vec{E}\right)$$

$$U' = -\mu\cdot\left(\vec{B} - \frac{\vec{v}}{c}\times\vec{E}\right)$$

$$e\vec{E} = -\frac{\vec{r}}{r}\frac{dV}{dr}$$

$$U' = \frac{-ge}{2mc}\vec{S}\cdot\vec{B} + \frac{g}{2m^2c^2}(\vec{S}\cdot\vec{L})\frac{1}{r}\frac{dV}{dr} \; ?$$

no electron rotating

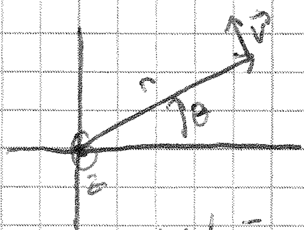
$$\left(\frac{d\vec{G}}{dt}\right)_{\text{non-rot}} = \left(\frac{d\vec{G}}{dt}\right)_{\text{rest frame}} + \vec{\omega}\times\vec{G}$$

$$\left(\frac{d\vec{S}}{dt}\right)_{\text{non-rot}} = \vec{S}\times\left(\frac{ge\vec{B}'}{2mc} - \omega_T\right)$$

$$U = \frac{-ge}{2mc}S\cdot\vec{B} + \vec{S}\cdot\vec{\omega}_T + \frac{g}{2m^2c^2}(\vec{S}\cdot\vec{L})\frac{1}{r}\frac{dV}{dr}$$

what is ω_T?

What is ω_T? Angular velocity of the precession of the frames in the laboratory rest frame.



$$\omega_T = -\frac{1}{2m^2c^2} L \frac{1}{r}\frac{\partial V}{\partial r}$$

$$U' = \frac{-ge}{2mc}\vec{S}\cdot\vec{B} + \frac{(g-1)}{2m^2c^2}(\vec{S}\cdot\vec{L})\frac{1}{r}\frac{\partial V}{\partial r} \quad \text{with } g=2$$

$$\begin{bmatrix} ds' \\ dt' \end{bmatrix} = \gamma \begin{bmatrix} 1 & -\frac{v}{c} \\ -\frac{v}{c} & 1 \end{bmatrix}\begin{bmatrix} s \\ t \end{bmatrix} \qquad A(-\beta) = \gamma\begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix}$$

$\gamma = (1-\frac{v^2}{c^2})^{-1/2}$

$\beta = \frac{v}{c}$

$\gamma = (1-\beta^2)^{-1/2}$

$$A(\beta+\delta\beta)A(-\beta) = \gamma(\beta+\delta\beta)\begin{bmatrix} 1 & -(\beta+\delta\beta) \\ -(\beta+\delta\beta) & 1 \end{bmatrix}\gamma(\beta)\begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix}$$

$$= \gamma(\beta+\delta\beta)\gamma(\beta) * \begin{bmatrix} 1-\beta(\beta+\delta\beta) & -\delta\beta \\ -\delta\beta & 1-\beta(\beta+\delta\beta) \end{bmatrix}$$

$$\Delta\Omega = \frac{\gamma^2}{\gamma+1}\beta\times\delta\beta \quad ?$$

$$\approx \frac{1}{1-\frac{v^2}{c^2}}\begin{bmatrix} 1-\frac{v^2}{c^2}-\beta\delta\beta & -\delta\beta \\ -\delta\beta & 1-\frac{v^2}{c^2}-\beta\delta\beta \end{bmatrix}$$

$$= \gamma^{-2}\begin{bmatrix} 1-\gamma^2\beta\delta\beta & -\gamma^2\delta\beta \\ -\gamma^2\delta\beta & 1-\gamma^2\beta\delta\beta \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 1 & -\gamma^2\delta\beta \\ -\gamma^2\delta\beta & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}\ \gamma^{-2}\begin{bmatrix} 1-\frac{1}{2}\gamma^2\delta(\beta^2) & -\gamma^2\delta\beta \\ -\gamma^2\delta\beta & 1-\frac{1}{2}\gamma^2\delta(\beta^2) \end{bmatrix}$$

$$= \begin{bmatrix} \gamma^{-2}-\frac{1}{2}\delta(\beta^2) & -\delta\beta \\ -\delta\beta & \gamma^{-2}-\frac{1}{2}\delta(\beta^2) \end{bmatrix} \approx \begin{bmatrix} \gamma^{-2} & -\delta\beta \\ -\delta\beta & \gamma^{-2} \end{bmatrix}$$

$(1-\frac{v^2}{c^2})$ vs $(1-\frac{v^2}{c^2})^{-1/2} \approx 1+\frac{1}{2}\frac{v^2}{c^2}$

$$ds'/\delta t = \frac{(1-\gamma^2)^{-1/2}}{\gamma^2}\frac{ds'}{\partial t'} \quad ??$$

Solution of the Radial Helium Atom ①

$$-\frac{\hbar^2}{2m_e}\nabla^2 - \frac{(2e_c)(e_c)}{4\pi\epsilon_0}\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_{12}}\left(\frac{1}{2}\right)\right) = \hat{H}, \quad E = \frac{1}{\Psi}\hat{H}\Psi$$ r_{12} is distance, 7C

with radial symmetry, $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}$. If $q_1 = \frac{1}{4\pi}\frac{1}{r_{10}^2}\delta(r_1) k_1^{-r_{10}}$, $q_2 = \frac{1}{4\pi}\frac{1}{r_{20}^2}\delta(r_2) k_2^{-r_{20}}$.

the energy stored is $\int q_1 \frac{1}{r_{12}} \frac{1}{4\pi\epsilon} q_2 \,\partial Vol_1 \partial Vol_2 = \frac{1}{4\pi\epsilon}\int q_1 \frac{1}{r_{12}} q_2 \,\partial Vol_1 \partial Vol_2$. Apply radial symmetry.

Apply the definition of voltage as $\int_r^\infty \vec{E}\cdot d\vec{l}$. Assume q_1 is "inside". $q_2 E_1 = \frac{k_1 k_2}{4\pi\epsilon}\frac{1}{r^2}$ $q_2\int_{r_2}^\infty E_1(r)\,\partial r = \frac{k_1 k_2}{4\pi\epsilon}\frac{1}{r_2}$

~~So $\int q_1 \frac{1}{r_{12}}\frac{1}{4\pi\epsilon} q_2 \,\partial Vol_1 \partial Vol_2 = -\int \frac{k_1}{4\pi r_{10}^2}\frac{k_2}{4\pi r_{20}^2}$ * - - -~~ $\nabla^2 V = -\rho/\epsilon$ $\int \rho_1 V_2 \,\partial Vol_1 \partial Vol_2 = \int \rho_1 \frac{\nabla^{-2}\rho_2}{\epsilon}$

$\int \rho_1 V_2 \,\partial Vol_1 = \int \rho_1 \nabla^{-2}\left(-\frac{1}{\epsilon}\right)\rho_2 \,\partial Vol_1 = \int \rho_1 \frac{1}{r_{12}}\frac{\rho_2}{4\pi\epsilon}\,\partial Vol_1 \partial Vol_2$

with $\nabla^{-2} f = \int_0^r \left(\frac{1}{r^2}\int_0^r r^2 f(r)\,\partial r\right)\partial r$ $\quad \int \rho_1 \rho_2 \frac{1}{r_{12}}\,\partial Vol_1 \partial Vol_2 = 4\pi \int \rho_1 \nabla^{-2}\rho_2 \,\partial Vol_1$

If the solutions for $\Psi_{\vec{r}_1,\vec{r}_2}$ can be separated then $\Psi(\vec{r}_1,\vec{r}_2) = \Psi(r_1)\Psi(r_2)$

So $\int \Psi \frac{1}{r_{12}}\,\partial Vol_1 \partial Vol_2 = \int \Psi(r_1)\frac{1}{r_{12}}\Psi(r_2)\,\partial Vol_1 \partial Vol_2 = 4\pi\int \Psi_1(r_1)\nabla^{-2}\Psi_2(r_2)\,\partial Vol_1$

$$\int \hat{H}\Psi \,\partial Vol_1 \partial Vol_2 = E\int \Psi \,\partial Vol_1 \partial Vol_2$$

Radial trial functions: $\Psi(\vec{r}_1,\vec{r}_2) = \left(\sum_{n=0}^{N} a_n r_1^n\right)\left(\sum_{m=0}^{M} b_m r_2^m\right) e^{-\lambda(r_1+r_2)}$

Since each sum term can be formed to represent an exponential function, this form is generic. It can be normalized or not with E being calculated as

$$E = \frac{\int \hat{H}\Psi \,\partial Vol_1 \partial Vol_2}{\int \Psi \,\partial Vol_1 \partial Vol_2}$$

Now, the interaction $\dfrac{\int \Psi(r_1)\frac{1}{r_{12}}\Psi(r_2)\,\partial Vol_1\,\partial Vol_2}{\int \Psi(r_1)\Psi(r_2)\,\partial Vol_1\,\partial Vol_2}$ needs calculation

$$\int \Psi(r_1)\frac{1}{r_{12}}\Psi(r_2)\,\partial Vol_1\,\partial Vol_2 = 4\pi\int \Psi_1(r_1)\nabla^{-2}\Psi(r_2)\,\partial Vol_1\,\cancel{\partial Vol_2}$$

$$\nabla^{-2}\Psi(r_2) = \int_0^{r_2}\left(\frac{1}{r^2}\int_0^r r^2 e^{-\alpha r}dr\right)dr = \int_0^{r_2}\left(\frac{C}{r^2} - \frac{2\exp(-\alpha r)}{\alpha^3 r^2} - \frac{2e^{-\alpha r}}{\alpha^2 r} - \frac{e^{-\alpha r}}{\alpha}\right)dr$$

The integrand needs to be finite as $r \to 0$ since the "change probability" is finite.

$$C = 2/\alpha^3$$

Confirmed by calculation(S) $\dfrac{\partial}{\partial r}\left(-\dfrac{2}{\alpha^3}\dfrac{1}{r} + \dfrac{2\exp(-\alpha r)}{\alpha^3 r} + \dfrac{\exp(-\alpha r)}{\alpha^2}\right)$

$$= \frac{2}{\alpha^3}\frac{1}{r^2} - \frac{2\exp(-\alpha r)}{\alpha^3 r^2} - \frac{2\exp(-\alpha r)}{\alpha^2 r} - \frac{\exp(-\alpha r)}{\alpha}$$

So(S) $\int \Psi_1 \nabla^{-2}\Psi_2 (4\pi)\,\partial Vol_1$?

$$= (4\pi)^2\int\left(\frac{2}{\alpha^3}\frac{1}{r^2} - \frac{2\exp(-\alpha r)}{\alpha^3 r^2} - \frac{2\exp(-\alpha r)}{\alpha^2 r} - \frac{\exp(-\alpha r)}{\alpha}\right)r^2 e^{-\alpha r}dr$$

$$= (4\pi)^2\left(\frac{7}{4}\frac{\exp(-2\alpha r)}{\alpha^4} - \frac{2\exp(-\alpha r)}{\alpha^4} + \frac{3r\exp(-2\alpha r)}{2\alpha^3} + \frac{r^2\exp(-2\alpha r)}{2\alpha^2}\right)\Bigg|_0^\infty$$

$$= (4\pi)^2\left(\frac{1}{4}\right)\frac{1}{\alpha^4}$$

$$\int \Psi_1\Psi_2\,\partial Vol_1\,\partial Vol_2 = (4\pi)^2\int r_1^2\exp(-\alpha r_1)\,\partial r_1\int r_2^2\exp(-\alpha r_2)\,\partial r_2 = (4\pi)^2\left(\frac{2}{\alpha^3}\right)^2$$

$$E'_{12} = \left((4\pi)^2\left(\frac{1}{4}\right)\frac{1}{\alpha^4}\right)\Big/\left((4\pi)^2\left(\frac{2}{\alpha^3}\right)^2\right) = \frac{1}{16}\alpha^2 \quad E_{12} = \frac{(2e_c)(e_c)}{4\pi\epsilon_0}\left(\frac{1}{2}\right)\left(\frac{1}{16}\alpha^2\right)$$

(S) Gigenmath software

This result has α^2 so this could not be correct. α needs to show in the first power.

Example ∅: Assume a trial function of the form $e^{-\alpha(r_1+r_2)}$.
Set α to be "hydrogen-like"

$$\left(-\frac{\hbar^2}{2m_e}\nabla^2 - \frac{(2e_c)(\cancel{2}e_c)}{4\pi\epsilon_0 r}\right)e^{-\alpha r} = \left(\left(\frac{-\hbar^2}{2m_e}\right)\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}\right)+\left(\frac{-(2e_c)(e_c)}{4\pi\epsilon_0}\frac{1}{r}\right)\right)e^{-\alpha r} = Ee^{-\alpha r}$$

$$\left(\frac{-\hbar^2}{2m_e}\right)\frac{\partial^2}{\partial r^2}e^{-\alpha r} = Ee^{-\alpha r} \Rightarrow E = \left(-\frac{\hbar^2}{2m_e}\right)\alpha^2$$

$$\left(\left(\frac{-\hbar^2}{2m_e}\right)\frac{2}{r}\frac{\partial}{\partial r} + \left(\frac{-(2e_c)(e_c)}{4\pi\epsilon_0}\right)\frac{1}{r}\right)e^{-\alpha r} = 0 \Rightarrow \left(\frac{-\hbar^2}{2m_e}\right)(-2\alpha) = \frac{(2e_c)(e_c)}{4\pi\epsilon_0}$$

$\alpha = \frac{2m_e}{\hbar^2}\left(\frac{1}{2}\right)\frac{(2e_c)(e_c)}{4\pi\epsilon_0}$. The Bohr radius[1] is $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{m_e c\alpha'}$

So $\alpha = \cancel{2/a_0}\ 2/a_0$ here for Helium (with +2 nuclear charge)

$$E = -\frac{2m_e}{\hbar^2}\left(\frac{e_c^2}{4\pi\epsilon_0}\right)^2$$

The Rydberg[2] Energy R_E is $R_E = \frac{1}{2}(m_e c^2)\alpha'^2 = 13.6\text{eV}$

$E_n = -\frac{Z^2}{n^2}R_E$

For Helium $E_n = -\frac{4}{n^2}R_E$. $E = -\frac{4}{n^2}$ Hartree[3] where 1 Hartree $\cong -13.6\text{eV}$ $= -13.59844\text{eV}$[4]

So now the zeroth approximate binding energy for Helium can be compared to the experimental 24.58741 eV.

The ionized state is $E_{ionized} = -4$ Hartree

Without counting the electronic interaction between electrons, in the ground state the electrons to the nucleus each one bound by −4 Hartree, giving −8 Hartree so far.

① Wikipedia "Bohr radius" $a_0 = 52.9177\times10^{-12}$ m ④ Wikipedia "Ionization energies of the elements"
② Wikipedia "Bohr model"

The interaction term is related to $\nabla^{-2}\psi(r_2)$

$$\nabla^{-2}\psi(r) = \frac{2}{\alpha^3}\frac{1}{r^2} - \frac{2e^{-\alpha r}}{\alpha^3 r^2} - \frac{2e^{-\alpha r}}{\alpha^2 r} - \frac{e^{-\alpha r}}{\alpha}$$

The interaction is $\dfrac{\int \psi(r_1)\frac{1}{r_{12}}\psi(r_2)\,\partial Vol_1\,\partial Vol_2}{\int \psi(r_1)\,\psi(r_2)\,\partial Vol_1\,\partial Vol_2} \overset{?}{=} (4\pi)\int \psi_1 \nabla^{-2}\psi_2\,\partial r_2\,\partial Vol_1$

On r_1, the equivalent "charge" acts centrally. That is $\frac{1}{4\pi\epsilon_0}\int_0^{r_1} 4\pi r_2^2\,\psi(r_2)\,\partial r_2$ with charge

So the interaction integral is $\int \psi(r_1)\frac{1}{r_{12}}\psi(r_2)\,\partial Vol_1\,\partial Vol_2 = \int \frac{1}{r_1}(4\pi)r_1^2 \int_0^{r_1}(4\pi)r_2^2\psi(r_2)\,\partial r_2\,\partial$

$$= (4\pi)^2\int_0^{\infty} r_1\left(\frac{2}{\alpha^3} - \frac{2e^{-\alpha r}}{\alpha^3} - \frac{2re^{-\alpha r}}{\alpha^2} - \frac{r^2e^{-\alpha r}}{\alpha}\right)\partial r$$

$$= (4\pi)^2\left(\frac{11}{8}\frac{e^{-2\alpha r}}{\alpha^5} - \frac{2e^{-\alpha r}}{\alpha^5} + \frac{11re^{-2\alpha r}}{4\alpha^4} - \frac{2re^{-\alpha r}}{\alpha^4} + \frac{7r^2e^{-2\alpha r}}{4\alpha^3} + \frac{r^3e^{-2\alpha r}}{2\alpha^2}\right)\Bigg|_0^{\infty}$$

$$= (4\pi)^2\frac{5}{8}\frac{1}{\alpha^5} \Rightarrow E_{12}' = \left((4\pi)^2\left(\frac{5}{8}\right)\frac{1}{\alpha^5}\right) \Big/ \left((4\pi)^2\left(\frac{2}{\alpha^3}\right)^2\right) = \frac{5}{32}\alpha$$

$$E_{12} = \frac{(2e_c)(e_c)}{4\pi\epsilon_0}\left(\frac{1}{2}\right)\left(\frac{5}{32}\right)\alpha = \frac{(e_c)^2}{4\pi\epsilon_0}\left(\frac{5}{32}\right)\left(\frac{2}{a_0}\right) = 8.54382\text{ eV}$$

If each electron's energy is raised by 8.54382 eV when bound this way, then the ionization energy for Helium would expect to be

$$4 * 13.59844 - 2 * 8.54382 = 37.3061\text{ eV}$$ versus the experimental 24.58741 eV

However, assuming complete shielding and compensating by calculating

$$E_{ionization} = 13.59844\text{ eV} + 8.54382\text{ eV} = 22.1423\text{ eV}$$

Comes very close to the experimental 24.58741 eV (within 10%)

$\vec{\nabla}\cdot\vec{B} = 0 \quad \Rightarrow \vec{B} = \vec{\nabla}\times\vec{A}$

$\frac{\partial}{\partial t}\vec{B} = \lim_{\Delta t\to 0} \vec{\nabla}\times\left(\vec{A}(t+\Delta t) - \vec{A}(t)\right)\frac{1}{\Delta t} = -\vec{\nabla}\times\vec{E}$ Form of curl source for $\vec{B}$

The difference has to be of the form $\vec{\nabla}\times(-\vec{E})$ because $\vec{B} = \vec{\nabla}\times\vec{A}$. Otherwise $\vec{\nabla}\cdot\vec{B}(t+\Delta t) \neq 0$,

$\vec{\nabla}\times(\vec{E} + \partial\vec{A}/\partial t) = \vec{0}$, $\vec{\nabla}\times\vec{E}_\otimes + \frac{\partial\vec{B}c}{\partial ct} = 0$ $\vec{\nabla}\cdot\vec{E}_\otimes = 0$

$(\vec{\nabla}\times, \frac{\partial}{\partial ct})\cdot(-\frac{\partial}{\partial ct}, \vec{\nabla}\times)c\vec{A}_\otimes = 0$ with $\vec{E}_\otimes = -\frac{\partial\vec{A}c}{\partial ct}$, $\vec{B} = \vec{\nabla}\times A$ $(\vec{\nabla}\times, \frac{\partial}{\partial ct})\cdot(\vec{E}_\otimes, c\vec{B}) = \vec{0}$

So what about the commute?

$(-\frac{\partial}{\partial ct}, \vec{\nabla}\times)\cdot(\vec{\nabla}\times, \frac{\partial}{\partial ct})\vec{A}_\otimes c = 0$ because $\vec{\nabla}\times$ and $\frac{\partial}{\partial ct}$ commute

If the stress is $(\vec{E}_\otimes, c\vec{B})$ then what is the strain?

$\vec{B} = \mu\vec{H}$ $\vec{D} = \epsilon\vec{E}$; in free space μ and ϵ are constant

$(\vec{\nabla}\times, \frac{\partial}{\partial ct})\cdot(\vec{\nabla}\times, \frac{\partial}{\partial ct})\vec{A}_\otimes c = c\mu\vec{J}_\otimes = (-\frac{\partial}{\partial ct}, \vec{\nabla}\times)\cdot(-\frac{\partial}{\partial ct}, \vec{\nabla}\times)\vec{A}_\otimes c = \vec{J}_\otimes\sqrt{\frac{\mu}{\epsilon_0}}$

$\vec{E} = -\vec{\nabla}V - \partial c\vec{A}/\partial ct$ $\vec{\nabla}\cdot\vec{E} = -\nabla^2 V - \frac{\partial c\,\vec{\nabla}\cdot\vec{A}}{\partial ct} = \rho/\epsilon$

If $\vec{\nabla}\cdot c\vec{A} + \frac{\partial V}{\partial ct} = 0 \rightarrow (\vec{\nabla}\cdot, \frac{\partial}{\partial ct})\cdot(\vec{A}, V) = 0 \Rightarrow (-\nabla^2 + \frac{\partial^2}{\partial ct^2})V = +\rho/\epsilon$

$(\nabla\times, \frac{\partial}{\partial ct})\cdot(\vec{\nabla}\times\frac{\partial}{\partial ct})\vec{A}_\otimes c = (\frac{\partial^2}{\partial ct^2} - \nabla^2)\vec{A}_\otimes c$, since $\vec{\nabla}\cdot\vec{A}_\otimes = 0$ Same form

$\vec{\nabla}\times(\hat{i}\hat{i} + \hat{j}\hat{j} + \hat{k}\hat{k}) = (\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z})\times(\hat{i}\hat{i} + \hat{j}\hat{j} + \hat{k}\hat{k})$

$= \frac{\partial}{\partial x}(\hat{k}\hat{j} - \hat{j}\hat{k}) + \frac{\partial}{\partial y}(\hat{i}\hat{k} - \hat{k}\hat{i}) + \frac{\partial}{\partial z}(\hat{j}\hat{i} - \hat{i}\hat{j})$

$= \hat{i}(\frac{\partial}{\partial y}\hat{k} - \frac{\partial}{\partial z}\hat{j}) + \hat{j}(\frac{\partial}{\partial z}\hat{i} - \frac{\partial}{\partial x}\hat{k}) + \hat{k}(\frac{\partial}{\partial x}\hat{j} - \frac{\partial}{\partial y}\hat{i})$

$= \hat{x}(\frac{\partial}{\partial y}\hat{z} - \frac{\partial}{\partial z}\hat{y}) + \hat{y}(\frac{\partial}{\partial z}\hat{x} - \frac{\partial}{\partial x}\hat{z}) + \hat{z}(\frac{\partial}{\partial x}\hat{y} - \frac{\partial}{\partial y}\hat{x}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \hat{x} & \hat{y} & \hat{z} \end{vmatrix}$

What about a 4-curl: $\begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z & \partial/\partial ct \\ \hat{x} & \hat{y} & \hat{z} & \hat{\tau} \\ \hat{x} & \hat{y} & \hat{z} & \hat{\tau} \\ \hat{x} & \hat{y} & \hat{z} & \hat{\tau} \end{vmatrix} = \frac{\partial}{\partial x}\hat{y}\hat{z}\hat{\tau}$ + anti-symmetric permutations

There are 6 permutations: $\hat{y}$ can go in 3 spots, $\hat{z}$ can go in 2 spots, and $\hat{\tau}$ in the remaining spot. Is each permutation associated with an $\vec{E}$ or $\vec{B}$ element

$-\hat{\tau}\,(\vec{\nabla}\times)(\hat{x}\hat{x} + \hat{y}\hat{y} + \hat{z}\hat{z})$ + antisymmetric permutations

$\vec{\nabla}\times\vec{E}_\otimes + \frac{\partial}{\partial ct}c\vec{B}_\otimes = 0$

$\vec{\nabla}\times c\vec{B}_\otimes - \frac{\partial}{\partial ct}\vec{E} = \vec{J}\sqrt{\frac{\mu}{\epsilon}}$

If so, it seems to make sense to have $\vec{B}_\otimes$ associated with $\frac{\partial}{\partial ct}\hat{z}\hat{y}\hat{x}$ + perms and have $\vec{E}_\otimes$ associated with $\frac{\partial}{\partial x}\hat{y}\hat{z}\hat{\tau}$ + antisymmetric permutations

$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \qquad \vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \vec{\nabla}\times\vec{E} + \frac{\partial c\vec{B}}{\partial ct} = 0$

$\vec{\nabla}\times\vec{H} - \frac{\partial}{\partial t}\vec{D} = \vec{J} \qquad \vec{\nabla}\cdot\vec{D} = \rho \Rightarrow \vec{B} = \mu\vec{H},\ \vec{D} = \epsilon\vec{E}$, μ,ϵ const $\vec{\nabla}\times c\vec{B} - \frac{\partial}{\partial ct}\vec{E} = \sqrt{\frac{\mu}{\epsilon}}\vec{J}$

$$\begin{bmatrix}\vec{\nabla}\times & -\frac{\partial}{\partial ct}\\ \frac{\partial}{\partial ct} & \vec{\nabla}\times\end{bmatrix}\begin{bmatrix}c\vec{B}\\ \vec{E}\end{bmatrix} = \sqrt{\frac{\mu}{\epsilon}}\begin{bmatrix}\vec{J}\\ \vec{\emptyset}\end{bmatrix} = \sqrt{\frac{\mu}{\epsilon}}\vec{J}\begin{bmatrix}1\\0\end{bmatrix},\quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon}$$

$$\vec{\nabla}\times = \left(\hat{i}\frac{\partial}{\partial x}\times + \hat{j}\frac{\partial}{\partial y}\times + \hat{k}\frac{\partial}{\partial z}\times\right)(\hat{i}\hat{i}\cdot + \hat{j}\hat{j}\cdot + \hat{k}\hat{k}\cdot)$$

$$= \frac{\partial}{\partial x}(\hat{k}\hat{j} - \hat{j}\hat{k})\cdot + \frac{\partial}{\partial y}(\hat{i}\hat{k} - \hat{k}\hat{i})\cdot + \frac{\partial}{\partial z}(\hat{j}\hat{i} - \hat{i}\hat{j})\cdot$$

$$= \hat{i}\left(\frac{\partial}{\partial y}\hat{k} - \frac{\partial}{\partial z}\hat{j}\right)\cdot + \hat{j}\left(\frac{\partial}{\partial z}\hat{i} - \frac{\partial}{\partial x}\hat{k}\right) + \hat{k}\left(\frac{\partial}{\partial x}\hat{j} - \frac{\partial}{\partial y}\hat{i}\right)$$

$$\begin{bmatrix}\frac{\partial}{\partial ct} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\end{bmatrix}\begin{bmatrix}0 & -E_x & -E_y & -E_z\\ E_x & 0 & -cB_z & cB_y\\ E_y & cB_z & 0 & -cB_x\\ E_z & -cB_y & cB_x & 0\end{bmatrix} = \begin{bmatrix}\rho/\epsilon\\ \sqrt{\frac{\mu}{\epsilon}}J_x\\ \sqrt{\frac{\mu}{\epsilon}}J_y\\ \sqrt{\frac{\mu}{\epsilon}}J_z\end{bmatrix}^T$$

Momentum is the stress
Mass and current is the strain

$\vec{E}\to c\vec{B},\ c\vec{B}\to -\vec{E}$, sources $\to 0$, $\vec{E} = \vec{x}_E E_x + \vec{y}_E E_y + \vec{z}_E E_z = \hat{e}E$, $c\vec{B} = \vec{x}_B cB_x + \vec{y}_B cB_y + \vec{z}_B cB_z = cB\hat{b}$

$\vec{\nabla}\times(\vec{E} + c\vec{B}) - \frac{\partial}{\partial ct}(\hat{b}\hat{e} - \hat{e}\hat{b})\cdot(\vec{E} + c\vec{B}) = \vec{J}$, $\vec{\nabla}_E\cdot(\vec{E} + c\vec{B}) = \rho/\epsilon = \vec{\nabla}\cdot(\vec{E} + c\vec{B})$

Here $\vec{J} = J_x\hat{x}_E + J_y\hat{y}_E + J_z\hat{z}_E \quad \vec{\nabla}_E = \frac{\partial}{\partial x}\hat{x}_E + \frac{\partial}{\partial y}\hat{y}_E + \frac{\partial}{\partial z}\hat{z}_E,\ \vec{\nabla} = \vec{\nabla}_E + \vec{\nabla}_B$

$$\begin{bmatrix}\frac{\partial}{\partial ct} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\end{bmatrix}\begin{bmatrix}0 & -cB_x & -cB_y & -cB_z\\ cB_x & 0 & E_z & E_y\\ cB_y & \cancel{E_z}\,-E_z & 0 & -E_x\\ cB_z & E_y & E_x & 0\end{bmatrix} = \vec{\emptyset}$$

$\vec{B} = \vec{\nabla}\times\vec{A}_\otimes$
$\vec{E}_\otimes = -\frac{\partial}{\partial t}\vec{A}_\otimes$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\vec{E}\cdot\vec{\nabla}\times\vec{H} + \vec{H}\cdot\vec{\nabla}\times\vec{E} = -\vec{E}\cdot\left(\vec{J} + \frac{\partial}{\partial t}\epsilon\vec{E}\right) + \vec{H}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \frac{1}{2}\frac{\partial}{\partial t}(\epsilon E^2 + \mu H^2) + \vec{E}\cdot\vec{J} = \frac{1}{2}\epsilon\frac{\partial}{\partial ct}(E^2 + cB^2) + \vec{E}\cdot\vec{J}$$

$\vec{\nabla}\times(c\vec{B} - j\vec{E}) - \frac{\partial}{\partial ct}(c\vec{B}j + \vec{E}) = \sqrt{\frac{\mu}{\epsilon}}\vec{J} \qquad \hat{\theta}\,\frac{1}{r}\frac{\partial}{\partial\theta} \times \varphi\hat{\theta} = -\frac{1}{r}\frac{\partial\varphi}{\partial\theta}\hat{r}$

$-j\vec{\nabla}\times(jc\vec{B}_\otimes + \vec{E}_\otimes) - \frac{\partial}{\partial ct}(jc\vec{B}_\otimes + \vec{E}) = \sqrt{\frac{\mu}{\epsilon}}\vec{J} \Rightarrow \sqrt{\frac{\mu}{\epsilon}}\vec{J}_\otimes + \frac{\partial}{\partial ct}\vec{E}_\otimes = 0$

Say $\vec{\varphi} = \varphi(\hat{i}\hat{i} + \hat{j}\hat{j} + \hat{k}\hat{k}) \Rightarrow \vec{\nabla}\times\vec{\varphi} = \hat{i}\left(\frac{\partial\varphi}{\partial y}\hat{k} - \frac{\partial\varphi}{\partial z}\hat{j}\right) + \hat{j}\left(\frac{\partial\varphi}{\partial z}\hat{i} - \frac{\partial\varphi}{\partial x}\hat{k}\right) + \hat{k}\left(\frac{\partial\varphi}{\partial x}\hat{j} - \frac{\partial\varphi}{\partial y}\hat{i}\right)$

Or $\vec{\varphi} = \varphi(x-y)\hat{k} + \varphi(y-z)\hat{i} + \varphi(z-x)\hat{j} \Rightarrow \vec{\nabla}\cdot\vec{\varphi} = \emptyset$

or $\vec{\varphi} = \varphi(r_{jk}, \theta_k)\hat{k} + \varphi(r_{ki}, \theta_i)\hat{i} + \varphi(r_{ij}, \theta_j)\hat{j} \Rightarrow \vec{\nabla}\cdot\vec{\varphi} = \emptyset$

$\vec{\nabla} = \frac{\partial A}{\partial u}\hat{u}$

$\frac{\partial}{\partial ct}A = \frac{\partial A}{\partial u}\frac{du}{\partial ct}$

$$\begin{bmatrix}\vec{\nabla}\times & -\frac{\partial}{\partial ct}\\ \frac{\partial}{\partial ct} & \vec{\nabla}\times\end{bmatrix}\begin{bmatrix}\vec{\nabla}\times\\ -\frac{\partial}{\partial tc}\end{bmatrix}c\vec{A}_\otimes = \sqrt{\frac{\mu}{\epsilon}}\begin{bmatrix}1\\0\end{bmatrix}\vec{J}_\otimes$$